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Note on the Figure of the Earth. By Captain Clarke, R.E.

The geodetical operations executed in India extend from 8° 10′ north latitude to 29° 30′; those in Europe from 38° 40′ to 70° 40′. The hitherto existing gap of nine degrees is now half filled up (though in the opposite hemisphere) by the valuable results of Mr. Maclear's measurement at the Cape, given in the last number of the *Monthly Notices* of the Royal Astronomical Society. I propose to examine the effect of this new arc in modifying the results obtained for the figure of the earth at page 771 of the *Account of the Principal Triangulation of Great Britain and Ireland*, referring to that work for the method of calculation. If we assume for the radius of curvature of the meridian the expression

$$\varrho = \mathrm{A} + 2\,\mathrm{B}\cos 2\lambda + 2\,\mathrm{C}\cos 4\lambda$$

the condition $5\,\mathrm{B}^2 - 6\,\mathrm{A}\,\mathrm{C} = 0$ makes the curve an ellipse; let

$$\frac{1}{\mathrm{A}} = \frac{1 + \dfrac{u}{10000}}{20890000}$$

$$-\frac{2\,\mathrm{B}}{\mathrm{A}} = \frac{1}{200} + \frac{v}{10000}$$

then if x be the necessary correction to the latitude of the north end, the corrections to the latitudes of the other stations will be as follows:—

Kamies Sector Berg..	$-8{\cdot}213 + 0{\cdot}2224\,u + 0{\cdot}1120\,v + x$
Heerenlogements Berg	$+0{\cdot}325 + 0{\cdot}8032\,u + 0{\cdot}3833\,v + x$
Royal Observatory ..	$-0{\cdot}676 + 1{\cdot}5104\,u + 0{\cdot}6746\,v + x$
Zwart Kop	$+0{\cdot}923 + 1{\cdot}6155\,u + 0{\cdot}7141\,v + x$
Cape Point	$-0{\cdot}229 + 1{\cdot}6608\,u + 0{\cdot}7308\,v + x$

From these numbers are obtained by multiplication the following,—

$(a\,m) = -1{\cdot}4757$ $\qquad \frac{1}{6}\,(a)\,(m) = -7{\cdot}6238$

$(b\,m) = -0{\cdot}7595$ $\qquad \frac{1}{6}\,(b)\,(m) = -3{\cdot}4297$

$(a^2) = +8{\cdot}3440$ $\qquad \frac{1}{6}\,(a)^2 = +5{\cdot}6305$

$(a\,b) = +3{\cdot}7190$ $\qquad \frac{1}{6}\,(a)\,(b) = +2{\cdot}5330$

$(b^2) = +1{\cdot}6585$ $\qquad \frac{1}{6}\,(b)^2 = +1{\cdot}1395$

The values of M A, M B, &c., (the differences of the above) resulting from this arc are as follows—the second line showing the values of these quantities for all the previously measured arcs:—

M A	M B	A²	A B	B²
+ 6·1481	+ 2·6702	2·7135	+ 1·1860	0·5190
143·4540	− 72·4048	282·0495	− 32·6817	56·3121

For the numbers in the second line see page 768 of the

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work quoted above. Adding together the corresponding quantities in the two lines, the following equation results,—

$$0 = +\ 149{\cdot}6021 + 284{\cdot}7630\,u - 31{\cdot}4957\,v$$
$$0 = -\ 69{\cdot}7346 - 31{\cdot}4957\,u + 56{\cdot}8311\,v$$

If we write P and Q for the absolute terms of this equation, it becomes by elimination

$$0 = u + {\cdot}0037410\,P + {\cdot}0020733\,Q$$
$$0 = v + {\cdot}0020733\,P + {\cdot}0187450\,Q$$

The numerical values of u and v are,—

$$x = -\ 0{\cdot}4151\,;\ v = +\ 0{\cdot}9970$$

from which we obtain the following results,—

$$a = 20926364$$
$$b = 20855340$$
$$a : b = 294{\cdot}63 : 293{\cdot}63$$

These semiaxes differ only by 16 and 107 feet respectively from the values obtained at page 771 of the *Account*, &c. The corrections to the latitudes in the Cape arc are,—

	″
North end	+ 1·280
Kamies Sector Berg	− 6·914
Heerenlogements Berg ..	+ 1·653
Royal Observatory	+ 0·649
Zwart Kop	+ 2·244
Cape Point	+ 1·090

Mr. Maclear states that the position of Kamies Berg station is such as fully to account for this large deflection. Let us therefore suppose it ascertained that the form of the adjoining country produces a deflection of n'' northwards, and trace the consequent effect upon the above values of a and b. The increments of $(a\,m)$ and $(b\,m)$ are,—

$$+\ {\cdot}2224\,n, \qquad +\ {\cdot}1120\,n,$$

and of $\frac{1}{6}\,(a)\,(m)$ and $\frac{1}{6}\,(b)\,(m)$ the corresponding increments are,—

$$+\ {\cdot}9687\,n, \qquad +\ {\cdot}4358\,n$$

and the increments of P and Q will be the differences of these, or,

$$-\ {\cdot}7463\,n, \qquad -\ {\cdot}3238\,n$$

from which

$$\delta u = +\ {\cdot}0035\,n : \delta v = +\ {\cdot}0076\,n$$

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and finally $\delta a = -2{\cdot}0\,n$; $\delta b = -12{\cdot}6\,n$. Let $n = 7$ then $\delta a = -14$; $\delta b = -88$. Applying these alterations to the values found for a and b, there result,

$$a = 20926350$$
$$b = 20855252$$
$$a : b = 294{\cdot}33 : 293{\cdot}33$$

These semiaxes exceed those given at p. 771 of the *Account*, &c., by 2 and 19 feet respectively. We have, therefore, this interesting result, that the very slight modification in the elements of the earth's figure and dimensions due to the arc at the Cape is entirely owing to the extraordinary disturbance of the vertical line at Kamies Berg.

On the Development of the Disturbing Function in the Lunar Theory. By A. Cayley, Esq.

The development of the disturbing function for the lunar theory is effected in a very elegant manner in Hansen's *Fundamenta Nova*, but a further step is requisite to exhibit the result in a perfectly explicit form and to compare it with those of other geometers. To do this is the immediate object of the present memoir, the mode of development being a reproduction of that made use of by Hansen. The results are compared with those obtained by Lubbock and Pontécoulant. But the memoir is written with a view to the development and application to the lunar theory of the theory contained in the author's "Memoir on the Problem of disturbed Elliptic Motion," and the notation is consequently that of the memoir just referred to.

The solar eclipse of Sept. 7, 1858, was observed at the Isle of St. Croix, Danish West Indies, by Major Lang. The last contact occurred at $21^{h}\ 9^{m}\ 18^{s}$ local mean time. The sky was free of clouds, but the direct heat of the sun's rays, about 130°, was almost insurmountable. No appearance of Baily's beads nor any other remarkable phenomenon. The atmosphere pure and clear. A very large spot with an extensive cluster on the sun's south-west quadrant, the whole length of which the day before was computed to be about 75,000 English miles.

The following note is by Mr. Williams, the Assistant-Secretary of the Society:—

"This morning, about twenty-five minutes past eleven, chancing to look at the sun, which was at that time greatly obscured by a fog, I distinctly saw without optical assistance